Meeting 5 Math 22

Activity 9.5.4 (continued)

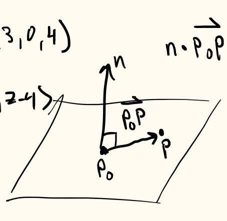
- Complete as a class.

(c) Two planes are parallel iff their normal vectors are parallel

$n = \langle 2, -1, 1\rangle$ $P_0 = (3, 0, 4)$ $n \cdot \overrightarrow{P_0P} = 0$

$0 = \langle 2,-1,1\rangle \cdot \langle x-3, y-0, z-4\rangle$

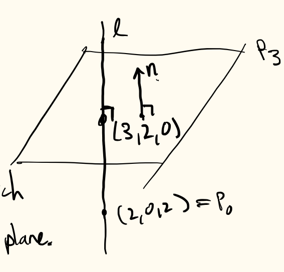
$= 2(x-3) - y + z - 4$



Simplify $2x - y + z = 10$

(d) P_3: $x + 2y - 2z = 7$

We need a point on the line $P_0 = (2, 0, 2)$ and a direction vector which is perpendicular to the plane.



We can take the direction to be $n = \langle 1, 2, -2\rangle$.

$r(t) = \overrightarrow{OP_0} + tn$

$= \langle 2,0,2\rangle + t\langle 1,2,-2\rangle$

$= \langle 2+t, 2t, 2-2t\rangle$ $x(t) = 2+t$, $y(t) = 2t$, $z(t) = 2-2t$.

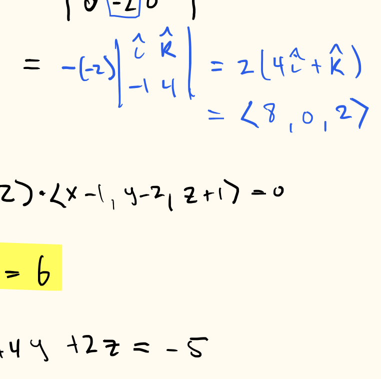
(e) We are looking for a value of t s.t.

$9t - 2 = (2+t) + 4t - 2(2-2t) = x(t) + 2y(t) - 2z(t) = 7$

$t = 1$ $r(1) = (3, 2, 0)$

Activity 9.5.5

- Complete 9.5.4 and discuss w/ your group
- Class discussion.



(a) $\overrightarrow{P_0P_1} = \langle 0, -2, 0\rangle$

$\overrightarrow{P_0P_2} = \langle -1, -1, 4\rangle$

(b) $n = \overrightarrow{P_0P_2} \times \overrightarrow{P_0P_1} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 4 \\ 0 & -2 & 0 \end{vmatrix}$

$= -(-2)\begin{vmatrix} \hat{i} & \hat{k} \\ -1 & 4 \end{vmatrix} = 2(4\hat{i} + \hat{k})$

$= \langle 8, 0, 2\rangle$

(c)

$8x - 8 + 2z + 2 = \langle 8,0,2\rangle \cdot \langle x-1, y-2, z+1\rangle = 0$

$8x + 2z = 6$

(d) q: $-3x + 4y + 2z = -5$

$m = \langle -3, 4, 2\rangle$ $P_1 = (0, 0, -5/2)$

$P_2 = (0, -\frac{5}{4}, 0)$

(e) $\theta = \cos^{-1}\left(\frac{n \cdot m}{|n||m|}\right) = \cos^{-1}\left(\frac{-24+4}{\sqrt{68}\cdot\sqrt{29}}\right) \approx 63^0$

Section 9.6 Vector-Valued Functions

Questions from Reading 3?

Activity 9.6.4

- Complete as a class.

Given $f(x,y) = x^2 + y^2$

Parabolas:

(a) $(x=2)$ $z = 4 + y^2$ $r_x(t) = \langle 2, t, 4+t^2\rangle$

(b) $(y=-1)$ $z = x^2 + 1$ $r_y(t) = \langle t, -1, t^2+1\rangle$

(c) $(z=25)$ $25 = x^2 + y^2$ a circle w/ radius 5

$r_z(t) = \langle 5\cos t, 5\sin t, 25\rangle$

(d) $(x=2)$ $z = 4 - y^2$ $S_x(t) = \langle 2, t, 4-t^2\rangle$

$(y=-1)$ $z = x^2 - 1$ $S_y(t) = \langle t, -1, t^2-1\rangle$

$(z=25)$ $25 = x^2 - y^2$ $S_z(t) = \langle 5\sec(t), 5\tan(t), 25\rangle$

$\tan^2 + 1 = \sec^2$

$1 = \sec^2 t - \tan^2 t$

End of 9.6

Section 9.7 Derivatives of Vector Valued Functions

Reading Debrief

- Discuss Activity 9.7.2 w/ your group.
- Questions?

Summary of Activity 9.7.2

(a) vector! Its the red arrow.

(b) vector! See Geogebra for picture.

(c)

It represents the average velocity of the particle as it moves from $r(t)$ to $r(t+h)$.

(d) This vector is tangent to the curve at the point $r(t)$. It represents instantaneous velocity of the particle. Its magnitude is the speed.

Section 9.7.2 Computing Derivatives

Since limits of vector-valued functions are computed component-wise, so are derivatives:

Derivative of a Vector-Valued function

If $r(t) = \langle x(t), y(t), z(t)\rangle$, then

$r'(t) = \langle x'(t), y'(t), z'(t)\rangle$

for all values of t at which x, y, z are differentiable.

Activity 9.7.3

- Complete w/ your group.
- Class discussion.

(a) $r'(t) = \langle -\sin(t), 1\cdot\sin(t) + t\cos(t), \frac{1}{t}\rangle$

(b) $r'(t) = \left\langle 2t+3, -2e^{-2t}, \frac{1\cdot(t^2+1) - 2t^2}{(t^2+1)^2}\right\rangle$

(c) $r'(t) = \langle \sec^2 x, -2t\sin(t^2), e^t + -te^{-t}\rangle$

(d) $r'(t) = \langle \frac{1}{2}(t^4+4)^{-1/2}\cdot 4t^3, 3\cos(3t), -4\sin(4t)\rangle$

Properties of the Derivative

Let f be a differentiable scalar function.
Let r, s be differentiable vector-valued functions. Then

(vector sum rule) 1. $\frac{d}{dt}(r(t) + s(t)) = \frac{d}{dt}r(t) + \frac{d}{dt}s(t)$

(vector product rule) 2. $\frac{d}{dt}(f(t)r(t)) = f'(t)r(t) + f(t)r'(t)$

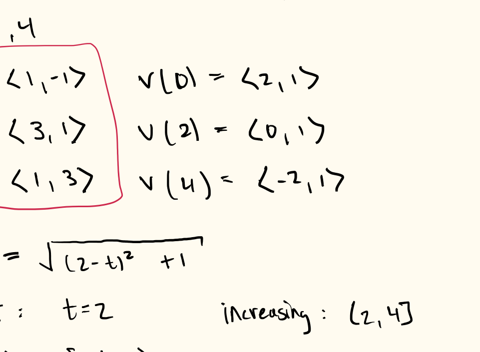
(scalar product rule) 3. $\frac{d}{dt}(r(t)\cdot s(t)) = r'(t)\cdot s(t) + r(t)\cdot s'(t)$

(vector product rule) 4. $\frac{d}{dt}(r(t)\times s(t)) = r'(t)\times s(t) + r(t)\times s'(t)$

(vector chain rule) 5. $\frac{d}{dt}(r(f(t))) = r'(f(t))\cdot f'(t)$

Activity 9.7.4

- Complete w/ your group.
- Class discussion



(a) $v(t) = r'(t) = \langle 2-t, 1\rangle$

(b) $a(t) = v'(t) = \langle -1, 0\rangle$

(c) $t = 0, 2, 4$

$r(0) = \langle 1, -1\rangle$ $v(0) = \langle 2, 1\rangle$

$r(2) = \langle 3, 1\rangle$ $v(2) = \langle 0, 1\rangle$

$r(4) = \langle 1, 3\rangle$ $v(4) = \langle -2, 1\rangle$

(d) $|v(t)| = \sqrt{(2-t)^2 + 1}$

Slowest: $t = 2$ increasing: $(2, 4]$

decreasing: $[-4, 2)$

(e) At $t=2$, $v(2)$, $a(2)$ are perp.

On $(2,4]$, $v(t), a(t)$ have acute angle

On $[-4, 2)$, $v(t), a(t)$ have obtuse angle.

(f) $\frac{d}{dt}(v\cdot v) = v'\cdot v + v\cdot v'$

$= 2(v\cdot a)$

Section 9.7.3 Tangent Lines

Every line is determined by a direction vector v and a point on the line. Suppose $r(t)$ is a vector function. Then $r'(t)$ is a vector that is tangent to the curve at the point $r(t)$.

The tangent line L to the curve through the point $r(a)$ is

$L(t) = r(a) + t\,r'(a)$

Activity 9.7.5

- Complete and discuss w/ your group
- Class discussion

(a) $r(\pi) = \langle \cos(\pi), -\sin(\pi), \pi\rangle$

$= \langle -1, 0, \pi\rangle$

(b) $r'(\pi) = \langle -\sin(\pi), -\cos(\pi), 1\rangle$

$= \langle 0, 1, 1\rangle$

(c) $L(t) = r(\pi) + t\,r'(\pi)$

$= \langle -1, t, t+\pi\rangle$